The ‘analysis’ of a century: Describe the influence of published works and the attitudes of their authors on the etymological development of the word ‘analysis’ in a mathematical context to 1750

A close assessment of William Oughtred’s *Clavis Mathimaticæ* of 1647, Isaac Newton’s *De analysi per aequationes numero terminorum infinitas* of 1669 and Leonard Euler’s *Introductio ad analysin infinitorum* of 1748 will be used to examine the influence of published works on the etymological development of the word ‘analysis’ in a mathematical context in the century spanning their publication.

This essay observes a gradual untethering of the word ‘analysis’ from its synthetic geometrical roots over the century. The analytic-synthetic distinction that this essay considers is what historians Otte and Panza term the ‘linguistic’ interpretation: “a mathematical argument or the formulation of a mathematical problem or proof is synthetic if it uses the language of classical geometry and of the theory of proportions. It is analytic if it uses the language of equations, functions or operations.”[1] Oughtred’s thinking, though predominantly algebraic, was still tied to geometry and his ‘analysis’ was motivated as a means by which ancient writers’ geometrical works could be understood. Newton’s *De analysi* made relations between continuously changing magnitudes the object of study and included infinite equations in a definition of ‘analysis’. However, Newton reverted to synthetic methods and this did much to stunt the development of analytic methods in England. Euler centred ‘analysis’ around functions in his *Introductio* and was the first to stress an explicit distinction between the methods of geometry and algebra.[2]

The authors’ attitudes towards analytic methods, their publication strategies, and the nature of their published works impacted the extent to which their definitions of ‘analysis’ were taken up. Despite the limited mathematical content of the *Clavis,* its nature as an accessible textbook and its illustrious support ensured the engagement of a generation of English scholars with Oughtred’s ‘analysis’. Newton was reluctant to publish *De analysi* until the emergence of a priority dispute, which limited the influence that the text had on the development of the word ‘analysis’, especially in light of the alignment of Newton’s publication strategy with his later wish to promote synthetic methods. Like Oughtred, Euler constructed a successful textbook, only the *Introductio* was far from limited in mathematical content. The fact that much of its contents and notation is familiar to students today is testament to its huge influence on the etymological development of the word ‘analysis’.

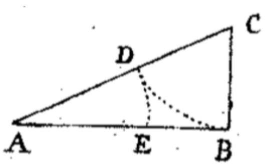
“Analyticall furniture is no lesse precious then plenteous”[3]: the demonstrations and legacy of Oughtred’s *Clavis*

In the wider mathematical setting of the *Clavis,* scholars in Northern Europe accessed classical mathematical texts, such as Pappus’ *Collectio,*[4] which stimulated them to make their own advancements.[5] In the narrower mathematical context, correspondences suggest that Oughtred had difficulty obtaining his European contemporaries’ texts. For example, he encountered Cavalieri’s methods in a letter from Paris, but was unable to purchase the original book.[6] However, Oughtred became familiar with Viète’s ‘analytic art’ through neighbour Charles Cavendish, who collected mathematical manuscripts from 1617.[7] It appears that Viète’s symbolical style so captured Oughtred’s imagination that he took as his mission the “inciting, assisting, and instructing [of] others”[8] in the ‘analytic art’. Oughtred’s status as a clergyman meant that he was reluctant to publish, but he agreed to publish the *Clavis* because it could be justified as the manual by which he taught the

[1] Otte & Panza, 1997, p.xi
[2] Ferraro, 2007, p.39
[3] Oughtred, 1647, p.68
[4] Bloye, 2015, p.v
[5] Stedall, 2015, p.33
[6] Rigaud, 1841, p.66
[7] Stedall, 2000, p.29
[8] Oughtred, 1632, p.9

Earl of Arundel's son.[9] Oughtred's motivation is explicit in the preface of the *Clavis*. He stated his aim "to direct [the readers] for the more easie and full understanding of the best and antientest Authors"[10] and explains that the *Clavis* is "not written in the usuall syntheticall manner … but in the inventive way of Analitice."[11] In summary, Oughtred was eager to teach 'analysis', as he understood it, to enable his readership to understand ancient writers' works.

To uncover the meaning of 'analysis' in the *Clavis*, I consider the contents of the 1647 edition. In the first chapter Oughtred defined the 'analytic art' as the process "in which by taking the things sought as knowne, we finde out that we seeke"[12] and explained that it shows "the processe of the whole worke: and so does not onely resolve the question in hand; but also teach a generall Theoreme."[13] The first ten chapters outlined the operations and notation that Oughtred went on to employ. The following chapters contained numerical and algebraic examples including binomial expansions (Oughtred's 'Potesates') to square and cube quantities, root extraction methods and equation-solving rules. Chapter eighteen, "The Analytical Store", supplied "Analyticall furniture" (algebraic expressions) which readers could employ, and were encouraged to invent, to solve problems. Chapter nineteen, at which content delivered previously "do[es] principally aime",[14] gave Euclidean propositions in an algebraic form. I consider the first problem of this chapter which is "the invention of [11 e 2]"[15]: the twelfth proposition of the second book of Euclid. This, as Oughtred explained, requires one "to cut B a Right line given, so that the Rectangle under the whole B, and the lesser segment, may be equall to the Quadrat of the greater Segment."[16] Oughtred split a line B into sections A (the longer) and $B - \mathsf{A}$, set up the equation "$\mathsf{Bq} - \mathsf{BA} = \mathsf{Aq}$"[17] and proceeded to solve for A, employing a result from the fourteenth chapter that considered the sums and differences of squares of algebraic quantities. Oughtred next illustrated that the solution is "geometrically effected thus."[18] He drew AB of length B and constructed CB perpendicular to AB of length $B/_{2}$. Utilising Pythagoras' Theorem, he obtained the length AC and subtracted $\mathsf{CD} = \mathsf{BC}$ to obtain the correct length for A by measuring $\mathsf{AE} = \mathsf{AD}$. For this problem he included the diagram below,[19] but some results from Euclid were expressed in purely algebraic form: a new departure in an elementary text.



Oughtred's geometrical construction

The *Clavis* taught that mathematics ought to be pursued 'analytically', which meant for Oughtred as for Viète that mathematical problems ought to be translated into symbolical equations and then solved algebraically. Whereas Viète had written philosophically of the 'analytic art', Oughtred tried to introduce it to young mathematicians.[20] Despite his use of at least 150 different symbols, his explanations had a geometric character. One concludes that Oughtred's 'analysis' was a powerful method for solving problems, predominantly geometrical problems, that left written traces[21] that

[9] Feingold, 2002, p.101
[10] Oughtred, 1647, preface
[11] Ibid.
[12] Ibid. p.4
[13] Ibid.
[14] Ibid. p.81
[15] Ibid.
[16] Ibid.
[17] Ibid. In modern notation: $B^2 - AB = A^2$
[18] Ibid.
[19] Ibid.
[20] Pycior, 1997, p.44
[21] Kaplan, 2018, 'Wallis's distinction: real characters against Greek sophistry'

could "teach a generall Theoreme." The importance ascribed to Euclid meant that 'analysis' was characterized as both forward- and backward-looking in the *Clavis*.

The *Clavis'* success in England cannot be attributed to the results that it contained but largely to its nature as an accessible textbook with no competitor.[22] In Europe most basic algebra texts were new editions of sixteenth-century works. Moreover, of the 24 mathematicians who published new books between 1600 and 1630, two-thirds wrote in their own language and the books in Latin lacked clarity of expression.[23] The *Clavis* filled the vacuity that existed between the poor quality or foreign language elementary texts and the inaccessible works of Viète and followers.[24] Its success was aided by its long list of illustrious supporters. For example, John Wallis, a pupil of Oughtred, insisted on the printing of the fourth and final editions and on its use well beyond the forty years it had been in circulation. Its value in teaching received recognition by Newton, who recommended it as a course of study at Christ's Hospital[25] and Edmund Halley, who remarked that it "may be of good use to all beginners in the Analytical Art."[26] Jonas Moore said "I owe all the Mathematicall knowledge I have [to the *Clavis*]."[27] Such accounts resulted in the *Clavis* being remembered long after the original grounds for its existence were forgotten.

Wielding a weapon or lifting a burden? The impact of changing attitudes to analytic mathematical methods in the works of Isaac Newton

Newton annotated books by Descartes, Viète, van Schooten, Oughtred and Wallis in the 1660s[28] and thus found himself at an intersection of new and old ideas about 'analysis'. The approach that Oughtred pioneered was continued in England by mathematicians such as John Pell and John Collins.[29] Young Newton saw himself as part of this tradition and his annotations to Oughtred and Viète and his support of the *Clavis* showed his interest. Newton's introduction to the new kind of 'analysis', which dealt with the infinite and the infinitesimal, was largely through Wallis, most notably through his 1656 *Arithmetica Infinitorum*.[30]

To understand Newton's concept of the new 'analysis', I consider the contents of *De analysi,* the key demonstrations of which include finding areas beneath curves; lengths of curves; infinite polynomials and their applications; the general binomial theorem; the numerical solution of equations and a foreshadowing of Newton's calculus. *De analysi* opened with rules detailing how to resolve quadrature problems: "rather briefly explained than narrowly demonstrated."[31]As is thematic, Newton stated that "the matter will be evident by example"[32] and guided readers through applications. Newton had discovered the general binomial theorem around 1664, drawing inspiration from Oughtred's "Analyticall Table"[33] and applying a Wallisian interpolation technique to obtain his infinite series. Despite having no rigorous process to justify the convergence of such series, Newton seldom erred. *De analysi* included a treatment of Newton's 'fluxions' and indeed, infinite series were inextricably linked with calculus for Newton, as indicated by their combination in Newton's 1671 treatise *Method of fluxions and infinite series*. At the end of *De analysi,* Newton noted: "two points stand out … as

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22 Stedall, 2000, p.37
23 Ibid. p.32
24 Ibid. p.36
25 Edleston, 1850, pp.279-292
26 As quoted in Stedall, 2002, p.85
27 Moore, 1650, Epistle to the Reader
28 Guicciardini, 2009b, p.4
29 Pycior, 1997, pp.70-102
30 Guicciardini, 2009b, p.144
31 Newton, 1669, p.207
32 Ibid. p.209
33 Oughtred, 1647, p.40

needing proof."[34] Newton justified his first quadrature rule using a new limit-style proof that makes the modern reader uncomfortable: he divides by the 'momentary increment' o and later lets "o to be zero."[35] Contrastingly, his second justification employs the help of the classical 'Eudoxean' axiom of the *Elements*.[36] This appeal to Euclid and the plethora of diagrams in the text renders the analytic-synthetic distinction in the text not quite clear.

Like the *Clavis,* the 'analysis' of *De analysi* is centred around equations but Newton placed the emphasis on relationships between continuously changing magnitudes[37] and extended equations to infinite equations: stating that he "never hesitated to bestow on [them] the name of analysis."[38] Whewell wondered "what manner of man [Newton] was who could wield as a weapon what we can hardly lift as a burden."[39] Initially, Newton thought his new 'analysis' to be a powerful "weapon". Some agreed: in 1745 Professor John Stewart went as far as explaining that the "Method of Exhaustions … [is] the first Step towards the general Method of Quadratures and of the converging Series lately introduced"[40]: rooting Newton's methods in undisputed classical ones. There were also doubts: Cantor observed that "[t]he [method] is just described, not proven."[41] Newton carried a "burden": he struggled to reconcile his philosophical agenda with his mathematical practice.[42] Printing algebraic, heuristic methods would expose Newton to criticism when he wanted certainty, and this was guaranteed by geometry. The algebraic 'analysis', he told Gregory, was "unfit to consign to writing and commit to posterity."[43] Newton structured his 1704 work on cubics, the *Enumeratio*, in a way that did not use analytic methods explicitly.[44] The *Principia* showcased his synthetic method of fluxions,[45] but in places he appealed to quadrature techniques that belonged to his new 'analysis'. This was noted by Montcula, who said: "the calculus surfaces through a concealment with which Newton hides it."[46] Those who understood Newton's proofs were frustrated. In reference to the impact of Newton's reversion to synthetic methods, Struik observed that "the tradition of the venerated Newton rested heavily upon English science."[47] Whilst many of Newton's followers sought, and struggled, to preserve a synthetic way of thinking,[48] Continental mathematicians had a fruitful period developing the new 'analysis',[49] of which Newton was originally a proponent.

An assessment of the significance of *De analysi* on the development of the word 'analysis' is complicated by the fact that it was only published in 1711, motivated in part by the priority dispute with Leibniz. Newton's teacher Barrow sent the work to philomath Collins in 1669, who could not convince Newton to publish.[50] This was typical of Newton's early works and stunted their impact. Most notably, in 1684 David Gregory sent his *Exercitatio geometrica de dimensione figurarum* to Newton,

[34] Newton, 1669, p.243
[35] Ibid.
[36] See Euclid's *Elements* Book V, Definition 5
[37] Fraser, 1997, p.56
[38] Ibid. p.241
[39] Whewell, 1837, p.167
[40] Stewart, 1745, p.346
[41] Cantor, 1901, p.104
[42] Domski, 2017, Introduction
[43] As quoted in Whiteside, 1961, p.196
[44] Guicciardini, 2009a, p.724
[45] Guicciardini, 2009b, p.16
[46] As quoted in Guicciardini, 2009a, p.721
[47] Struik, 1948, p.130
[48] Ibid.
[49] Bloye, 2015, p.158
[50] Guicciardini, 2004, p.457

which showed Gregory had discovered several series quadrature theorems found in *De analysi*.[51] As historian Love remarks, Newton's strategy of circulating his texts to a small circle of mathematicians can be described as "scribal publication", which contains the idea that "the power to be gained from the text was dependent upon possession of it being denied to others."[52] This strategy ensured that Newton had control over the dissemination of his ideas but caused problems: his career was riddled with priority disputes and arguably late eighteenth-century European mathematics would have been different had Newton published his innovative infinite series earlier.[53]

Euler's *Introductio*: The manifesto of a new mathematical discipline?

Euler was a "mathematical omnivore"[54] and his exploration of much of the mathematics existing at the time gave him a unique view of the architecture of the subject. This led to his identification of opportunities for generalisation. In eighteenth-century Europe calculus was viewed as both useful and vulnerable: there were particular concerns with the employment of infinite and infinitesimal quantities. For example, Euler warned that the methods in Newton's early works were " in danger of plunging into manifest contradiction."[55] Euler's preface to the *Introductio* explained that, to eradicate the "manifest contradiction" that he found in Newton and others' 'analysis', he would introduce topics "absolutely required for analysis"[56] so that the reader "almost imperceptibly becomes acquainted with the idea of the infinite."[57]

The *Introductio's* focus is apparent from the first chapter's title: "On Functions in General."[58] For Euler, functions were the objects of 'analysis' and the first chapter aimed to introduce and account for them.[59] Euler began with definitions, including that: "A function of a variable quantity is an analytic expression composed in any way whatsoever of the variable quantity and numbers or constant quantities."[60] Here, Euler's "analytic expression" is a formula constructed from variables in finitely many steps using algebraic and transcendental operations and composition of functions.[61] D'Alembert's 1747 derivation of the wave equation forced Euler to reassess what kind of functions were permissible. Most importantly, though, is Euler's attempt to move to a general theory and the change from curves to functions as the objects of 'analysis'. The generality of Euler's functions inspired their application. Indeed, for most of the nineteenth century the definition of a function as an expression was still present, for example in Eduard Heine's 1872 *Die Elemente der Functionenlehre*,[62] though by now multiple values had been eliminated.[63]

Euler turned to "the development of Functions in Infinite Series"[64] in chapter four. Euler motivated this by stating that infinite series are the "best form for the mind to grasp [the] nature [of

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[51] Guicciardini, 2004, p.464
[52] Love, 1993, p.184
[53] Guicciardini, 2004, p.456
[54] du Sautoy, 2010
[55] As quoted in du Sautoy, 2010
[56] Ibid.
[57] Ibid.
[58] Euler, 1748, p.2
[59] Panzo, 2007, p.4
[60] Euler, 1748, p.3
[61] Fraser, 1989, p.325
[62] *'Definition: A single-valued function of a variable x is an expression, which is unambiguously defined for every rational or irrational value of x'* Heine, 1872, p.180
[63] Stedall, 2015, p.255
[64] Euler, 1748, p.50

functions].”[65] Euler attempted to convince the reader that any function could be expressed by an infinite series by showing how to expand any algebraic function as well as various transcendental functions into such a series. His methods for algebraic functions were not new, being a combination of Newton’s methods using division and the binomial theorem.[66] The power of series methods in Euler’s hands is revealed in the seventh chapter: “Exponentials and Logarithms Expressed through Series.”[67] His techniques would now be frowned upon as they utilised infinite and infinitesimal quantities. Despite this, like Newton, he rarely erred. In §114,[68] he noted that since $a^0 = 1$, it follows that $a^\omega = 1 + \Psi$, where both ω and Ψ are infinitely small and so that “with a as the base for the logarithm, we have $\omega = \log(1 + k\omega)$.” He later noted that, for any j, $a^{j\omega} = (1 + k\omega)^j$, and employed the binomial theorem to get a series for $a^{j\omega}$. He let $j = \frac{z}{\omega}$ and noted that as “z denotes any finite number, since ω is infinitely small, then j is infinitely large.”[69] The series then became:

$$a^z = 1 + \frac{1}{1}kz + \frac{1(j-1)}{1 \cdot 2j}k^2z^2 + \frac{1(j-1)(j-2)}{1 \cdot 2j \cdot 3j}k^3z^3 + \cdots$$

Noting that “since j is infinitely large, $\frac{j-1}{j} = 1$”[70] and by similar logic, Euler arrived at

$$a^z = 1 + \frac{kz}{1} + \frac{k^2z^2}{1 \cdot 2} + \frac{k^3z^3}{1 \cdot 2 \cdot 3} + \cdots$$

where k depends on the base a. In §119,[71] by applying the binomial theorem, he derived an infinite series for $\log(1 + x)$. In §122[72] he does what the modern reader anticipates: the base of his logarithm is chosen so that $k = 1$, and the symbol e is introduced for the base of his “natural logarithm.”[73] It is hard to imagine a mathematics course on ‘analysis’ today without these concepts.

Also significant in the *Introductio* is the derivation of the power series for sine and cosine, starting from the identity $sin^2(x) + cos^2(x) = 1$. Before this, there was no sense of the trigonometric functions being expressed as formulas involving letters and numbers, whose relationship to other such formulas could be studied.[74] Euler unified elementary functions: he substituted $z = iv$ and $z = -iv$ into the identity $(1 + \frac{z}{j})^j$ derived at the end of the seventh chapter to obtain $\frac{e^{iv}+e^{-iv}}{2} = \cos v$ and $\frac{e^{iv}-e^{-iv}}{2i} = \sin v$ and so illustrated how trigonometric functions were related to the exponential function just introduced.

One must consider that Euler had no notion of a limit or of an infinitesimal approximation and did not justify all of his conclusions in formal terms or offer a proof for every result. For example, he applied Newton’s generalized binomial theorem without providing a proof for it.[75] Nevertheless, the *Introductio* should be praised highly for its structural character, the impressive range and quality of its results, and its achievement in uniting fundamental mathematical material. Whilst quantifying the

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[65] Ibid.
[66] Katz, 2009, pp.618-619
[67] Euler, 1748, p.85
[68] Ibid. p.92
[69] Ibid. p.93
[70] Ibid.
[71] Ibid. p.95
[72] Ibid. p.97
[73] Ibid.
[74] Katz, 2009, p.583
[75] Panzo, 2007, p.4

influence of the *Introductio* is difficult, the book was reprinted several times in the eighteenth century and was translated into French and German[76] and it is doubtful that any other essentially didactic work includes as large a portion of original material which survives in the mathematics courses of today.[77]

Conclusion

Oughtred's *Clavis* demonstrated that his symbolical 'analysis' was "precious" as an illustrative method to solve (predominantly geometric) problems to its readership and its impact on their notion of 'analysis' was "plenteous": more as a result of its success as a textbook than as a result of it being a radical mathematical work. The success of the *Clavis* ensured that Oughtred's successors harboured few worries about the legitimacy of the symbolical style and also that there would no longer be an exclusively geometric tradition in English mathematics; there would also be a parallel analytic one.

Newton was initially keen to continue the approach to 'analysis' that Oughtred pioneered and in his youth expanded the definition of 'analysis' to include methods that dealt with infinite and infinitesimal quantities. His initial disinclination to publish limited the influence that *De analysi* could have had on shaping the meaning of 'analysis'. His later reversion to geometry, and the structuring of his publication strategy around this agenda, convinced some mathematicians, especially in England, that Oughtred's symbolical approach was unfit for publication and that synthetic methods were to be employed to guarantee certainty. By the second half of the eighteenth century this had led to a divide between the 'analysis' found in Europe that was in the process of divorcing its geometrical roots[78] and the attempt to preserve synthetic methods in England.

Euler sought to eradicate the "manifest contradiction" that he found in the analytic methods of young Newton and others with his textbook, the *Introductio.* Euler's 'analysis' comes close to the modern discipline, that is the study of functions by means of infinite processes, especially through infinite series. Euler restructured and unified mathematics by placing 'analysis', which was devoid of diagrams and entirely distinct from geometry, at its heart. It is for these reasons that Euler has been referred to as "analysis incarnate"[79] and that the *Introductio* can be considered as the manifesto of 'analysis' as an autonomous mathematical discipline.

[76] Katz, 2007, p.232
[77] Boyer, 1951, p.225
[78] Guicciardini, 2004, pp.218–256
[79] Bell, 1937, Chapter 9

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